Online Scheduling Algorithm for Heterogeneous Distributed Machine Learning Jobs

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Abstract—Distributed machine learning (ML) has played a key role in today's proliferation of AI services. A typical model of distributed ML is to partition training datasets over multiple worker nodes to update model parameters in parallel, adopting a *parameter server* or *AllReduce* architecture. ML training jobs are typically resource elastic, completed using various time lengths with different resource configurations. A fundamental problem in a distributed ML cluster is how to explore the demand elasticity of ML jobs and schedule them with different resource configurations, such that the utilization of resources is maximized and average job completion time is minimized. To address it, we propose an online scheduling algorithm to decide the execution time window, the number and the type of concurrent workers and parameter servers for each job upon its arrival, with a goal of minimizing the weighted average completion time. Our online algorithm consists of (i) an online scheduling framework that groups unprocessed ML training jobs into a batch iteratively, and (ii) a batch scheduling algorithm that configures each ML job to maximize the total weight of scheduled jobs in the current iteration. Our online algorithm guarantees a good parameterized competitive ratio with polynomial time complexity. Extensive evaluations using real-world data demonstrate that it outperforms state-of-the-art schedulers in today's AI cloud systems.

Index Terms—Distributed Machine Learning; Online Scheduling

1 Introduction

Nowadays, most leading IT companies operate distributed machine learning (ML) clusters of GPU servers, to run ML jobs that train models over large datasets for providing AI-driven services. To train a large model, hundreds of concurrent workers (typically implemented on virtual machines or containers) are deployed in parallel. Either the training dataset or the ML model is partitioned among workers, realizing *data parallelism* or *model parallelism* [1][2][3]. In model parallelism, each worker updates part of the parameters using the entire input dataset [4]. In data parallelism, each worker has an entire copy of the ML model and computes parameter update (gradients) using a portion of input data; in each training iteration, workers exchange locally-computed gradients to obtain the global ML model update. As training data is usually enormous, data parallelism is the dominant form of parallel training in practice [1][3].

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There are two typical approaches for exchanging parameter updates among workers: parameter server (PS) framework and AllReduce framework [3][5]. In the PS framework, PSs maintain model parameters as a global key-value store, and each worker uploads computed gradients to the PSs. The PSs update the corresponding parameters based on received gradients and then send updated parameters to the workers. In the AllReduce framework, all nodes act as PS and worker concurrently and first exchange gradients with others to obtain the mean of the gradients. Then each node uses the resulting gradient to update the model parameters. The workers and PSs may be placed on different physical servers, when they cannot be completely accommodated on the same server, or to fully utilize expensive and fragment resources on servers [4].

ML training jobs are resource-intensive and time-consuming. Existing distributed ML systems [6][7][8] require job owners to estimate the amount of resources, including the number of workers and the resource configuration of each worker, as well as the time needed, to train the ML model using a large dataset. For example, Google uses Borg [9], and Microsoft, Tencent, and Baidu both use customized versions of YARN schedulers [8] to aggressively provision each job as much resource as possible according to user demand and job priority, using strategies such as FIFO and max-min fair allocations.

However, the job owner is often uncertain of the amount of resources and time it may take to complete a job. There is *elasticity* in ML jobs' resource demand: It takes different amounts of time to train a certain model with workers of different resource configurations, especially of different numbers of GPUs. Further, the processing time of a mini-batch is typically not inversely proportional to the amount of resource allocated to the worker, which is mainly due to overhead in parallel training [10]. Next, assigning training jobs less resources than what they require in the ideal case (*i.e.,* that leads to most expedited single-job training [10] [11] [12]) may reduce average training completion time in the entire system. For example, when training CIFAR-10 CNN for 100K steps until the model achieves 87% accuracy, the single-step training time (time to train a mini-batch) can be 15 milliseconds with a single GPU and 10 milliseconds with two GPUs (suppose it is the ideal case) [10]. Thus, if there are two training jobs of this type submitted at the same time and only three GPUs are available, with adequate other resources, allocating one GPU to one job and two GPUs for the other is the best strategy for minimizing the average job completion time, which results in $(10+15)/2 = 12.5$ milliseconds, in contrast to allocating two GPUs to each job sequentially, which results in $(10+20)/2 = 15$ milliseconds.

Considering demand elasticity, a fundamental problem for a ML cluster operator is: *Given limited resources, how to decide the number/type of workers (and PSs) and running time of each job, such that resources are maximally utilized and average weighted completion time is minimized?* Here, the weight of each job may characterize its processing priority.

To address the above problem, we first formulate the average weighted completion time minimization problem into a time-indexed mathematical program. The program formulates features of ML jobs (demand for large-volume data analysis capacity and high inter-node connection bandwidth). Different from traditional makespan minimization problems, it contains both conventional (packing-type) constraints and non-conventional (set-type and natural language described) constraints, which cannot be handled by existing approaches [13] [14]. Decision variables include the number/type of workers (and PSs), and the execution window of each job. To compute schedules on the go with the shortest completion time, we divide our design into two steps:

First, we propose an online framework to convert the online optimization problem into a series of batch scheduling problems by partitioning the overall timespan into intervals with geometrically increasing length. Our online scheduling framework employs a *dual approximation algorithm* as a subroutine for performance guarantee. The dual approximation algorithm finds an infeasible solution that is super-optimal, where the performance of the algorithm is measured by the degree of infeasibility allowed. The infeasible solution will finally become feasible as job execution can span multiple intervals. The super-optimal objective value contributes to bound the average weighted completion time. This dual algorithm is realized through a batch scheduling algorithm that solves the *maximum weighted schedule problem* to schedule as many unscheduled jobs as possible before a certain time point.

Second, we observe that the maximum weighted schedule problem includes several non-conventional constraints for characterizing the configuration/placement of workers and PSs. To handle these *set-type* and *natural language described* constraints, we encode each valid schedule in a variable and reformulate the original program into an integer linear program (ILP), where only conventional packing constraints are included, at the price of introducing an exponential number of variables. Instead of solving the ILP directly, which is infeasible in practice due to time complexity, we design an approximation algorithm by applying a tailored primal-dual framework to the ILP's LP relaxation and its dual LP. We interpret dual variables as unit resource prices, and compute the best schedule for each job based on resource consumption cost and its ML framework. The algorithm schedules a job if its weight is higher than its estimated serving cost.

We carry out rigorous theoretical analysis to prove that our online algorithm runs in polynomial time, and achieves a bounded competitive ratio. We evaluate practical effectiveness of our online algorithm through trace-driven simulation studies. We implement four representative job scheduling strategies used in existing cloud platforms, and compare them with our algorithm. Simulation results confirm that our algorithm outperforms existing methods by at least 30% in average weighted completion time, especially in systems with resource shortage.

In the following sections, we review related work literature in Sec. 2 and model the distributed ML system working with PS framework in Sec. 3. Sec. 4 present the online scheduling framework. Sec. 5 propose approximation algorithms for scheduling batch jobs. And Sec. 6 show that our online scheduling framework is also applicable to the distributed ML clusters working with Ring-AllReduce architecture. Simulation studies are presented in Sec. 7. Sec. 8 concludes the paper.

2 Related Work

Job Scheduling and Resource Allocation in Distributed ML Systems. Ghodsi *et al.* [6] propose a fair allocation policy of multiple resource types, similar to Mesos [7] and YARN [8]. In these systems, job owner prescribes the number and resource configuration of workers. In comparison, we design an online algorithm to guide worker deployment and resource allocation, exploiting the demand elasticity of ML jobs. Bao *et al.* [15] propose a deep learning-based job placement algorithm to minimize interference among co-located ML jobs. Resource allocation among multiple jobs is not considered by these work. Considering the heterogeneity of hardware accelerators and workloads, Narayanan *et al.* [16] propose Gavel, which expresses the existing scheduling policies as optimization problems, and uses a round-based scheduling mechanism. Xiao *et al.* [17] present AntMan, which co-designed cluster scheduler and deep learning framework. AntMan introduces dynamic scaling mechanisms for memory and computation to share GPU resources. It allows GPUs to be utilized by over-provision of opportunistic jobs at best-effort to minimize the interference between jobs. Jeon *et al.* [18] analyze the trace of deep learning jobs running on a multi-tenant GPU cluster in Microsoft, and study three factors that affect cluster utilization: job scheduling, locality on GPU utilization, and failures during training. Focusing on the fairness of GPU allocation, Mahajan *et al.* [19] design an ML scheduling framework, Themis, to achieve long term finish-time fairness. Themis presents a two-level scheduling architecture where ML apps can bid on resources offered in an auction. Gu *et al.* [20] propose a preemptive scheduler, Tiresias, which aims to minimize the average job completion time (*i.e.,* time from job submission to job completion). Tiresias assigns jobs according to the multiplication of a job's remaining workload and the number of resources, *e.g.*, GPUs, RAM and CPUs. The above schedulers study the scheduling problem of ML jobs, but they pay more attention to analyzing different characteristics of ML jobs, *e.g.,* the fairness or the heterogeneity of hardware. We explore the demand elasticity of ML jobs to maximally utilize resources, meanwhile minimize the average weighted job completion time. Amiri *et al.* [21] propose a centralized scheduling strategy that assigns tasks to workers to minimize the average completion time with the help of one master. Similarly, Yan *et al.* [22] develop performance models that quantify the impact of data partitioning and system provisioning on system performance and scalability. Above papers don't consider online job scheduling and resource sharing problems. Peng *et al.* [23] propose an online scheduler based on deep reinforcement learning to minimize the average job completion time. They dynamically adjust the number of worker/PS, but not the type. Bao *et al.* [4] design an online algorithm to guide resource allocation over time in a distributed machine learning system. Although we consider a similar problem, this work is significantly different from [4]. First, our work is the first that explores the demand elasticity. A job's scheduling and configuration are needed to be determined, while [4] focuses on adjusting the number of customized workers in each time slot, but does not address choices of different types of workers/PSs for a job, nor colocation of workers and PSs on the same physical server(s). Second, considering the demand elasticity of ML jobs, the goal of our work is to minimize the weighted completion time, while [4] aims to maximize the overall utility. Third, with the different optimization objective, our algorithmic idea to solve the weighted completion time minimization problem is also different from [4], as shown in Fig. 1.

Job Scheduling and Resource Allocation in Cloud Systems. Shi *et al.* [24] propose the first online combinatorial auction for cloud resource allocation and pricing. Zhang *et al.* [25] study online resource allocation in a cloud computing platform through posted-price mechanisms. Zhang *et al.* [26] design mechanisms for online cloud resource bundling and provisioning to maximize social welfare with server costs. Jiao *et al.* [27, 28] devise online prediction-free and prediction-aware algorithms to provision resources across clouds and edges for serving dynamic demands. These studies satisfy each job's demand within a fixed window, and do not consider the demand elasticity and scheduling dimensions in the solution space.

For job scheduling, Azar *et al.* [29] study online cloud job scheduling problems for deadline-sensitive jobs, assuming that one server can only execute one job in each time slot. Zhou *et al.* [30] design a mechanism for online cloud job scheduling and resource allocation, where jobs have alternative deadlines corresponding to different job valuations. Wang *et al.* [31] schedule jobs online via creating and running multiple replicas of each task in order to mitigate the straggler issue. The resource demand of each job is specified by the job owner in advance in the above literatures.

Resource Allocation in Other Systems. Sheikhalishahi *et al.* [32] study an open shop scheduling problem, considering the objective of human error, availability and make span. They apply three meta-heuristic methods to find the preferred solution. Tian *et al.* [33] design a scheduling framework to resolve co-flow scheduling of multi-stage jobs. Wang *et al.* [34] develop a co-flow scheduling system which focuses on minimizing the average weighted co-flow completion time. The scheduling problem studied in the above work only focus on resource constrains, and don't take the characteristics of PS framework into consideration.

3 System Model

3.1 System Overview

We consider a machine learning cluster where multiple ML training jobs run using potentially different ML frameworks (*e.g.*, TensorFlow [35], MXNet [1], CNTK [36]).

Especially, a set of *J* training jobs arrive with large input datasets during a large time span $[T] = 1, 2, ..., T$, to train different ML models using synchronous training, *i.e.*, synchronous stochastic gradient descent (S-SGD) method. Synchronous training can typically ensure model convergence and achieve higher model accuracy than asynchronous training [22][37], and is hence widely adopted over the latter in AI clouds of leading IT companies [38]. The large input dataset of job j ($j \in [J]$) is divided into D_j equal-sized data chunks. Each data chunk is divided into K_j equal-sized mini-batches. We consider two distributed ML architectures in this work:PS framework [3] and Ring-AllReduce architecture [5][39].

Let H denote the number of physical servers for the deployment of workers and PSs. Each server $h \in [H]$ offers C_h^r units of type-*r* resource. *R* represents the number of resource types, including GPU, CPU, memory and bandwidth [40][41]. Workers and PSs are implemented as virtual machines (VMs) or containers in physical servers. We refer to workers and PSs with different resource allocations as different types. Let M and P denote the number of worker and PS types, respectively. Each type-*m* ($m \in [M]$) worker (type-*p* ($p \in [P]$) PS) consumes e_m^r (z_p^r) units of type-*r* ($r \in [R]$) resource. Let b_m (B_p) be the bandwidth occupied by each worker *m* (PS p), *i.e.*, $b_m = e_m^{bandwidth}$ ($B_p = z_p^{bandwidth}$).

Upon the arrival of an ML job *j* at time a_j, the following decisions are made: (i) when to start the job, denoted by binary variable x_{jt}: $x_{jt} = 1$ if job *j* is executed with starting time *t*; (ii) the number of allocated type-*m* workers serving job *j* deployed on physical server *h* at and after a_j, indicated by integer variable y_{jhm}; (iii) the number of allocated type-*p* PSs serving job *j* deployed on physical server *h* at and after a_j, indicated by integer variable s_{jhp}; (iv) the amount of consecutive time slots allocated to job *j*, which is related to the number and processing capacity of workers serving job *j*, specified by d_j. We do not consider preemption in this work, because when a job is suspended, the entire image of the job needs to be stored temporarily, which increases the overhead. Table 1 summarizes important notations for easy reference.

3.2 Training Process with PS framework

The set of global parameters of each ML job is partitioned into several partitions, each maintained by one PS [3]. Each worker of job *j* has a complete replica of the training

TABLE 1: List of Notations

J	# of jobs	R	# of resource types
T	system timespan	$[X]$	interger set $\{1, 2, \ldots, X\}$
a_j	arrival time of j	D_j	# of data chunks in j
w_j	weight of job j	d_j	running duration of j
M	# of worker types	P	# of PS types
E_j	# of training epochs for job j		
K_j	# of mini-batches in one data chunk of job j		
H	# of servers to deploy workers and PSs		
C_h^r	capacity of type-r resource on server h		
$e_m^r(z_p^r)$	type-r resource of worker m (PS p)		
$b_m(B_p)$	bandwidth of worker m (PS p)		
v_{jm}	time to train a mini-batch of job j in worker m		
π_j	size of gradients generated by each worker after processing one mini-batch when serve job j		
U_j^p	time to update parameters at a type-p PS in each iteration of j		
ρ_{jm}^p	processing capacity of each worker when j employs worker m and PS p		
q_j	whether j's all workers (and PSs) are running in one server or not		
x_{jt}	whether or not training job j with starting time t		
s_{jhp}	# of type-p PSs serving job j in server h		
y_{jhm}	# of type-m workers serving job j in server h		

model. Each worker processes allocated mini-batches one by one, sends computed gradients to and receives updated parameters from all job j's PSs after processing one mini-batch (one iteration). The training process at all workers is synchronized: in each iteration, each PS updates its parameters after it has aggregated gradients from all workers, and then sends updated parameters to all workers. When the entire input dataset is trained for one round, an epoch is completed. For an ML job, the input dataset is trained for multiple epochs. Let E_j be the required training epochs of job j.

Let v_{jm} denote the time for a type-m worker to train a mini-batch of job j. Assume the computation time at a type-p PS for updating a partition of global parameters using gradients from all workers in each iteration of job j is a constant, indicated by U_j^p. The time for a type-m worker of job j, deployed on a server with no PS, to transfer gradients to all PSs in other servers is $\frac{\pi_j}{b_m}$, and vice versa, assuming the upload and download bandwidth are the same. When a worker is placed together with some PS(s) in one server, exchanging parameters/gradients with PS(s) in the same server needs no inter-server bandwidth and takes less time. With synchronous training, the time for exchanging gradients/parameters in one iteration of a job depends on the worker that spends the longest time, which is bound by $\frac{\pi_j}{b_m}$, *i.e.*, the time if any worker is not co-located with any PS.

We ignore fetching time of the input data as it can be largely hidden behind training using pipelining. Let q_j indicate whether all workers and PSs of job j are deployed in the same physical server (1) or not (0). Let ρ_{jm}^p denote the processing capacity of each worker, *i.e.*, the number of mini-batches that can be trained by each worker in one time slot, when job j employs type-m worker(s) and type-p PS(s). Thus, we have:

$$\rho_{jm}^p = \begin{cases} 1/(v_{jm} + U_j^p), & \text{if } q_j = 1 \\ 1/(v_{jm} + U_j^p + \dfrac{2\pi_j}{b_m}), & \text{if } q_j = 0 \end{cases} \tag{1}$$

Note that when not all workers and PSs of job j are on the same server ($q_j = 0$), ρ_{jm}^p represents the upper-bound of time for exchanging gradients/parameters in one training iteration, for model simplification.

3.3 Problem Formulation

We exploit the demand elasticity of ML jobs to minimize the sum of all jobs' weighted completion times [13], that is $\sum_{j \in J} w_j c_j$, where c_j denotes the completion time of job j and $c_j = \sum_{t \in [T]} x_{jt}(t + d_j)$, and w_j can be interpreted as the priority of job j [9]. The objective is equivalent to minimizing average weighted job completion time, given the fixed total number of jobs, J. In practice, a cluster manager can set job weights according to job arrival times, deadlines and workloads. Jobs, which have larger workload and smaller time interval between arrival time and deadline, can be assigned larger weights. The larger a job's weight is, the sooner it is scheduled. If all weights are the same, the system prefers to schedule small jobs earlier, as the total completion time is shorter. This discriminates large jobs. Assigning a larger weight to large jobs can mitigate the problem.

The offline minimization problem can be formulated as the following time-indexed program:

$$\text{minimize} \quad \sum_{j \in [J]} w_j \sum_{t \in [T]} x_{jt}(t + d_j) \tag{2}$$

subject to:

$$\sum_{t \in [T]} x_{jt} = 1, \forall j, \tag{2a}$$

$$|\{m \in [M] | \sum_{h \in [H]} y_{jhm} > 0\}| = 1, \forall j \tag{2b}$$

$$|\{p \in [P] | \sum_{h \in [H]} s_{jhp} > 0\}| = 1, \forall j \tag{2c}$$

$$q_j = 1 \text{ if and only if } h = h', \forall h, h' : y_{jhm} > 0, s_{jh'p} > 0, \forall j, \tag{2d}$$

$$\sum_{h \in [H]} \sum_{p \in [P]} s_{jhp} \geq 1, \forall j, \tag{2e}$$

$$d_j \sum_{h \in [H]} \sum_{m \in [M]} y_{jhm} \rho_{jm}^p \geq E_j D_j K_j, \forall j, \forall p : \sum_{h \in [H]} s_{jhp} > 0 \tag{2f}$$

$$\sum_{h \in [H]} \sum_{m \in [M]} y_{jhm} \leq D_j, \forall j, \tag{2g}$$

$$\sum_{j : t' \in (t - d_j, t]} x_{jt'} (\sum_{m \in [M]} e_m^r y_{jhm} + \sum_{p \in [P]} z_p^r s_{jhp}) \leq C_h^r, \forall t, \forall r, \forall h, \tag{2h}$$

$$\sum_{h' \in [H^{-h}]} \sum_{m \in [M]} y_{jh'm} b_m \leq \sum_{p \in [P]} s_{jhp} B_p, \forall j, \forall h : \sum_{p \in [P]} s_{jhp} > 0, \tag{2i}$$

$$x_{jt} = 0, \forall j, \forall t < a_j, \tag{2j}$$

$$y_{jhm} \in \{0, 1, ...\}, \forall j, \forall h, \forall m, \tag{2k}$$

$$s_{jhp} \in \{0, 1, ...\}, \forall j, \forall h, \forall p, \tag{2l}$$

$$d_j \in \{0, 1, ...\}, \forall j, \tag{2m}$$

$$x_{jt} \in \{0, 1\}, \forall j, \forall t. \tag{2n}$$

$$q_j \in \{0, 1\}, \forall j. \tag{2o}$$

where $\forall j, t, r, h, m, p$ represents $\forall j \in [J], t \in [T], r \in [R], h \in [H], m \in [M], p \in [P]$. Constraint (2a) requires job j to be scheduled once. Constraint (2b) ensures that each job selects and employs one type of workers, as it is common to use the same type of workers to process evenly allocated input data batches for synchronous training. Though there have been recent studies that assign different workers different batch sizes [42], the relevant study is still in its infancy and not widely used in practice. If different types of workers are used in a job, the time for the workers to process equal-sized data batches varies; hence, workers requiring less training time need to wait for slower workers in each iteration, leading to lower resource efficiency. Constraint (2c) requires that each job uses one type of PSs due to the same reason.

Constraint (2d) shows the relationship among q_j, y_{jhm} and s_{jhp}, which is hard and awkward to describe by linear constraint. Constraint (2e) assures that there is at least one PS allocated to each ML job for maintaining its global parameters. Constraint (2f) guarantees that for job *j*, a sufficient number of workers and time slots are allocated to accomplish training of the dataset for E_j epochs. $E_j D_j K_j$ is the total count of mini-batches trained in job *j*. Constraint (2g) upper-bounds the number of workers by the number of data chunks D_j, to ensure that one data chunk is trained by at most one worker for E_j epochs. The resource capacity of physical servers for running workers and PSs is formulated by constraint (2h). Here, $x_{jt'} = 1, t' \in (t - d_j, t]$ denotes that job *j* is still running in time slot *t*. Since each of job *j*'s workers needs to push gradients to and pull computed parameters from all its PSs, the bandwidth reservation for PSs of job *j* in server *h* should cover the total bandwidth of job *j*'s workers placed on other servers, which can be formulated as the linear constraint (2i). Here, H^{-h} represents the set of all the servers except h. Constraint (2j) indicates that it is impossible to start job *j* before its arrival.

Without the non-linear constraints (2b)(2d), the weighted completion time minimization problem in (2) is still a mixed integer linear program (MILP). Even in the offline setting, with information of all jobs given, solving such MILPs is non-trivial and typically NP-hard [43].

3.4 Algorithmic Idea

In order to solve the weighted completion time minimization problem, we design an efficient online algorithm with bounded competitive ratio (*i.e.*, the maximum ratio of the total weighted completion time incurred by our online algorithm over that incurred by the offline optimal approach which knows all the inputs in advance) in two steps, as shown in Fig. 1.

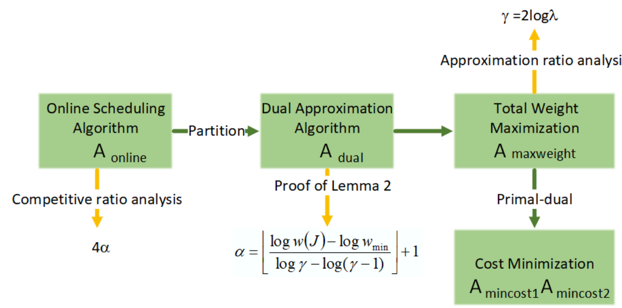

Fig. 1: Main idea of our online algorithm A_{online}.

i. In Sec. 4, we first group unprocessed ML jobs until a certain time point into a batch, to convert the online optimization problem into a series of batch scheduling problems. Then, we invoke a dual approximation algorithm A_{dual} to schedule jobs in a batch. According to Lemma 1 [14], the schedule produced by A_{dual} is required to satisfy two properties. It is hard to yield such a schedule directly. Rather than solving the the batch scheduling problem directly, we focus on a more solvable problem instead, *i.e.*, the total weight maximization problem. Leveraging an approximation algorithm $A_{maxweight}$ for the total weight maximization problem, A_{dual} constructs a required schedule.
ii. In Sec. 5, we introduce an approximation algorithm $A_{maxweight}$ for batch processing, which solves the the total weight maximization problem. $A_{maxweight}$ applies the primal-dual framework and employs two subroutines ($A_{mincost1}$ and $A_{mincost2}$) to choose the schedule with smallest cost for each job.

Here, A_{dual} is a subroutine of A_{online} and a dual approximation algorithm to solve the maximum weighted schedule problem in Definition 1. A_{dual} invokes $A_{maxweight}$ and $A_{maxweight}$ invokes $A_{mincost1}$ and $A_{mincost2}$. $A_{mincost1}$ and $A_{mincost2}$ solve the cost minimization problem in Sec. 5.2. Performance guarantees of various proposed algorithms are shown at the end of the yellow arrows in Fig. 1.

4 Online Scheduling Framework

In Sec. 4.1, we introduce an online scheduling framework A_{online} that partitions the timespan to group ML jobs. It requires a *dual approximation algorithm* A_{dual} for job scheduling, which is presented in Sec. 4.2.

4.1 Online Scheduling Algorithm

Our online algorithm is partly inspired by Leslie *et al.* [14]. The basic idea is to partition the timespan of potential completion times at geometrically increasing points, and iteratively schedule unprocessed ML jobs until a certain time point. More specifically, let $\tau_0 = 1$, $\tau_i = 2^{i-1}$. In rounds $i = 1, 2, \ldots$, we wait until time τ_i. Let J_i represent the set of jobs that have arrived by time τ_i, but still not scheduled. Next, we require a *dual approximation algorithm* A_{dual} for J_i, which produces a schedule of length at most $\alpha\tau_i$ ($\alpha > 1$, which is a number to indicate the infeasibility of the schedule produced by A_{dual}) and whose total weight is at least the optimal weight of the maximum weighted schedule problem in Sec. 5. The schedule generated by A_{dual} is then assigned to run from time $\alpha\tau_i$ to time $\alpha\tau_{i+1}$. Because $\alpha\tau_{i+1} - \alpha\tau_i \geq \alpha\tau_i$, it is flexible to run job with length at most $\alpha\tau_i$ in interval $[\alpha\tau_i, \alpha\tau_{i+1}]$, and hence our online algorithm produces feasible schedules.

Definition 1. The Maximum Weighted Schedule Problem: *In an ML cluster, given a deadline* τ_i*, a set of jobs* J_i *at the beginning, and a weight for each job, we aim to construct a feasible schedule that maximizes the total weight of jobs completed by time* τ_i.

In A_{online} (Algorithm 1), J_i^s denotes the set of jobs scheduled during round i. Note that $\tau_0 = 1$ implies the assumption that no job can complete within the first time slot. Lines 3-5 group unscheduled jobs into set J_i. We invoke the dual approximation algorithm Algorithm A_{dual} for J_i in line 6. Next, we run $j \in [J_i^s]$ from time $\alpha\tau_i$ to time $\alpha\tau_{i+1}$ according to the schedule produced by A_{dual} in line 8-9. In line 11, we add job(s) in J_i which is (are) not scheduled in round i to set J_{i+1}, to process in next round $i+1$.

Algorithm 1 An Online Algorithm A_{online}

Input: $T, C_h^r, \forall h \in [H], r \in [R]$;
Output: $x_{jt}, y_{jhm}, s_{jhp}, d_j, \forall j \in [J], t \in [T], m \in [M], p \in [P], h \in [H]$;
1: Initialize $x_{jt} = 0$, $y_{jhm} = 0$,$s_{jhp} = 0$, $d_j = 0$, $\forall j \in [J], t \in [T], m \in [M], p \in [P], h \in [H]$, $J_i = \varnothing$;
2: **while** $i = 1, 2, ...$ **do**
3:   **while** $t < \tau_i$ **do**
4:     $J_i = J_i \cup \{j\}$;
5:   **end while**
6:   $\{\{x_{jt}\}, d_j, \{y_{jhm}\}, \{s_{jhp}\}\}_{j \in J_i, t \in [\alpha\tau_i]} = A_{dual}(J_i, \tau_i, \{C_h^r\})$;
7:   **for all** $j \in [J_i^s]$ **do**
8:     Run job j from time $\alpha\tau_i$ to time $\alpha\tau_{i+1}$ according to $(\{x_{jt}\}, d_j, \{y_{jhm}\}, \{s_{jhp}\})$;
9:   **end for**
10:   $J_{i+1} = J_{i+1} \cup (J_i \setminus J_i^s)$;
11: **end while**

Lemma 1. *Given a dual approximation algorithm for* $J_i, i \in 1, 2, ...$, *which produces a schedule satisfying two properties: (i) the length of the schedule is at most* $\alpha\tau_i$*; (ii) total weight of the schedule is at least the optimal weight of the corresponding maximum weighted schedule problem,* A_{online} *is an online* 4α*-approximation algorithm to minimize the total weighted completion time.*

Proof: Consider a fixed optimal schedule for the problem in (2). Let I be chosen to be the smallest integer so that all jobs complete in this schedule by time τ_I, and let J_i^* denote the set of jobs that complete in the ith interval, $(\tau_{i-1}, \tau_i]$, $i = 1, 2, ..., I$. In a particular interval i, consider jobs completed during the first i intervals according to the optimal schedule, but do not run within the first $i-1$ iterations by A_{online}, *i.e.*, $J_i' = \cup_{k=1}^{i} J_k^* - (\cup_{k=1}^{i-1} J_k^s)$. Each job $j \in J_i'$ arrives by τ_i, since it can complete by τ_i in the optimal schedule, besides, it has not been scheduled before τ_i using A_{online}. That is $j \in J_i$, so that $J_i' \subset J_i$. Moreover, all jobs in J_i' can be scheduled to complete within τ_i by the optimal schedule for the total weighted completion times minimization problem, as well as the optimal solution of the maximum scheduled weight problem for J_i. According to the property (ii) of the dual approximation algorithm, we obtain a set J_i^s of total weight at least $\sum_{j \in J_i'} w_j$ in iteration i, *i.e.*, $w(J_i^s) \geq w(J_i')$, here $w(J) = \sum_{j \in J} w_j$. Furthermore, combining the definition of J_i', for each $i = 1, 2, ..., I$, the following inequation is satisfied: $\sum_{k=1}^{i} w(J_k^s) \geq \sum_{k=1}^{i} w(J_k^*)$.

It can be derived from the above inequation that A_{online} has scheduled all jobs by iteration I, *i.e.* $\sum_{i=1}^{I} w(J_i^s) = \sum_{i=1}^{I} w(J_i^*)$. Focus on the optimal schedule for the total weight completion time minimization problem, $\sum_{j \in [J]} w_j c_j^* \geq \sum_{i=1}^{I} \tau_{i-1} w(J_i^*)$. Here c_j^* denotes the completion time of job j in the optimal schedule. The schedule which is iteratively constructed by A_{online} has total weighted completion time at most $\sum_{i=1}^{I} \alpha\tau_{i+1} w(J_i^s) \leq 4\alpha \sum_{i=1}^{I} \tau_{i-1} w(J_i^s) \leq 4\alpha \sum_{i=1}^{I} \tau_{i-1} w(J_i^*) \leq 4\alpha \sum_{j \in [J]} w_j c_j^*$.

4.2 A Dual Approximation Algorithm

The dual approximation algorithm A_{dual} (Algorithm 2) produces desired schedules based on a γ-approximation algorithm for the Maximum Weighted Schedule Problem, that schedules as many unscheduled jobs as possible before a deadline (to be detailed in Sec. 5). Lines 2-4 invoke the γ-approximation algorithm $A_{maxweight}$ for α rounds. Specifically, in the ιth ($\iota \in [\alpha]$) round, we schedule jobs in $J_i \setminus J_i^s$, *i.e.*, jobs in J_i but not served in before rounds, from time $(\iota - 1)\tau_i + 1$ to time $\iota\tau_i$.

Lemma 2. *Given a* γ*-approximation algorithm for the maximum weighted schedule problem which schedules as many jobs as possible before deadline* τ_i*,* A_{dual} *constructs a schedule of length at most* $\alpha\tau_i$ *and total weight at least the optimal objective value of the corresponding maximum weighted schedule problem.*

Proof: Let $J_{i\iota}^*$ and $J_{i\iota}^s$ be the set of jobs served optimally and completed by A_{dual} in the ιth round, respectively. Thus, the optimal objective value of the total weight maximization problem for J_i is $w(J_{i1}^*)$. And let $J_{i\iota}^{s'} = J_{i\iota}^s \cap J_{i1}^*$. In the ιth round, the input of the γ-approximation algorithm is $J_i - \cup_{\iota'=1}^{\iota-1} J_{i\iota'}^s$. When $\iota = 1$, we have

$$w(J_{i1}^s) \geq \frac{1}{\gamma} w(J_{i1}^*). \tag{3}$$

For $\iota \geq 2$, consider jobs which can be scheduled by the optimal solution but are not served by A_{dual} in the first $\iota - 1$ rounds, *i.e.*, $J_{i1}^* - \cup_{\iota'=1}^{\iota-1} w(J_{i\iota'}^{s'})$. In ιth round, since each $j \in [J_{i1}^* - \cup_{\iota'=1}^{\iota-1} w(J_{i\iota'}^{s'})]$ can be completed by the optimal solution, $w(J_{i\iota}^*) \geq w(J_{i1}^* - \cup_{\iota'=1}^{\iota-1} w(J_{i\iota'}^{s'}))$. Then we have

$$w(J_{i\iota}^s) \geq \frac{1}{\gamma}(w(J_{i1}^*) - \sum_{\iota'=1}^{\iota-1} w(J_{i\iota'}^{s'})) \geq \frac{1}{\gamma}(w(J_{i1}^*) - \sum_{\iota'=1}^{\iota-1} w(J_{i\iota'}^{s})) \tag{4}$$

For $\iota \in [\alpha]$, the following inequality holds:

$$\sum_{\iota'=1}^{\iota} w(J_{i\iota}^s) \geq [1 - (1 - \frac{1}{\gamma})^{\iota}] w(J_{i1}^*) \tag{5}$$

We prove (5) by induction. (5) must hold for $\iota = 1$, since (3) holds. Suppose (5) holds for ι, according to (4), we have $\sum_{\iota'=1}^{\iota+1} w(J_{i\iota}^s) \geq \frac{1}{\gamma} w(J_{i1}^*) + (1 - \frac{1}{\gamma}) \sum_{\iota'=1}^{\iota} w(J_{i\iota}^s) \geq [1 - (1 - \frac{1}{\gamma})^{\iota+1}] w(J_{i1}^*)$. Thus we prove (5). Suppose for the specific ι^*, $\sum_{\iota'=1}^{\iota^*} w(J_{i\iota'}^s) \geq w(J_{i1}^*)$ and $\sum_{\iota'=1}^{\iota^*-1} w(J_{i\iota'}^s) < w(J_{i1}^*)$. Note that $J_{i1}^* - \cup_{\iota'=1}^{\iota^*-1} w(J_{i\iota'}^s) \neq \varnothing$, then $w(J_{i\iota^*}^s) \geq \min_{j \in [J_{i1}^*]} w_j \geq w_{min}$, here $w_{min} = \min_{j \in [J]} w_j$. And since (5), $w(J_{i\iota^*}^s) \geq (1 - \frac{1}{\gamma})^{\iota^*-1} w(J_{i1}^*)$. So $(1 - \frac{1}{\gamma})^{\iota^*-1} w(J_{i1}^*) \geq w_{min}$, then $\iota^* \leq \frac{\log w(J_{i1}^*) - \log w_{min}}{\log\gamma - \log(\gamma-1)} + 1$. We can set $\alpha = \lfloor \frac{\log w(J) - \log w_{min}}{\log\gamma - \log(\gamma-1)} \rfloor + 1$, which satisfies

$$\alpha \geq \lfloor \frac{\log w(J_{i1}^*) - \log w_{min}}{\log\gamma - \log(\gamma - 1)} \rfloor + 1 \geq \iota^*, \forall i \tag{6}$$

such that $\sum_{\iota'=1}^{\alpha} w(J_{i\iota'}^s) \geq w(J_{i1}^*), \forall i$.

Algorithm 2 A Dual Approximation Algorithm A_{dual}

Input: $J_i, \tau_i, C_h^r, \forall h \in [H], r \in [R]$;
Output: $x_{jt}, y_{jhm}, s_{jhp}, d_j, J_i^s, \forall j \in [J_i], t \in [\tau_i], m \in [M], p \in [P], h \in [H]$;
Initialize $x_{jt} = 0, d_j = 0, y_{jhm} = 0, s_{jhp} = 0, \beta_h^r(t) = 0, J_i^s = \varnothing, \delta_h^r(t) = \Delta_h^r(0), \forall j \in [J_i], t \in [\tau_i], m \in [M], h \in [H], p \in [P], r \in [R]$;
for $\iota = 1$ to α **do**
  $\{\{x_{jt}\}, d_j, \{y_{jhm}\}, \{s_{jhp}\}\}_{j \in (J_i \setminus J_i^s), t \in [(\iota-1)\tau_i+1, \iota\tau_i]} = A_{maxweight}(J_i \setminus J_i^s, \tau_i, \{C_h^r\})$;
end for

5 Approximation Algorithm for Total Weight Maximization

We next present an approximation algorithm $A_{maxweight}$ for batch processing, employing a primal-dual algorithm in Sec. 5.1. As subroutines of $A_{maxweight}$, we design two algorithms in Sec. 5.2 to compute the best schedule for each job. Theoretical analysis is presented in Sec. 5.3.

5.1 The Maximum Weighted Schedule Problem

We formulate a maximum weighted schedule problem for each round i in our online scheduling framework, that maximizes the total weight of jobs in J_i completed by time τ_i.

$$\text{maximize} \sum_{j \in [J_i]} \sum_{t \in [\tau_i]} w_j x_{jt} \tag{7}$$

subject to:

$$\sum_{t \in [\tau_i]} x_{jt} \le 1, \forall j \in [J_i], \tag{7a}$$

$$\sum_{t \in [\tau_i]} x_{jt}(t + d_j) \le \tau_i, \forall j \in [J_i], \tag{7b}$$

$$(2b) - (2i), (2k) - (2o), where\ \forall t \in [\tau_i].$$

This maximization problem involves integer variables, non-linear constraint (2b) (2c) and constraints concerning multiplication of variables (2f)(2h)(7b). To address these challenges, we first apply the compact-exponential techniques [30] to reformulate problem (7) into an equivalent conventional integer linear program (ILP) with packing structure:

$$\text{maximize} \sum_{j \in [J_i]} \sum_{l \in \Gamma_j} w_j x_{jl} \tag{8}$$

subject to:

$$\sum_{j \in [J_i]} \sum_{l: t \in T(l), h \in l} x_{jl} f_{jh}^r(l) \le C_h^r, \forall t \in [\tau_i], r \in [R], h \in [H], \tag{8a}$$

$$\sum_{l \in \Gamma_j} x_{jl} \le 1, \forall j \in [J_i], \tag{8b}$$

$$x_{jl} \in \{0, 1\}, \forall j \in [J_i], l \in \Gamma_j. \tag{8c}$$

In the above ILP, Γ_j is the set of feasible schedules for job *j*, each corresponding to the set of decisions $(x_{jt}, d_j, y_{jhm}, s_{jhp}, q_j, \forall m \in [M], p \in [P], h \in [H], t \in [\tau_i])$ satisfying constraints (7b)(2b)(2c)(2f)(2i)(2k)(2n). Binary variable x_{jl} indicates whether job *j* is scheduled according to schedule $l \in \Gamma_j$ or not, $\forall j \in [J], l \in \Gamma_j$. $T(l)$ records the allocated time slots of job *j* in schedule $l \in \Gamma_j$. We use $h \in l$ to indicate that schedule *l* uses server *h* to deploy workers and PSs for job *j*. $f_{jh}^r(l)$ denotes the total type-*r* resource occupation of job *j*'s schedule *l* on server *h*, *i.e.*, $f_{jh}^r(l) = \sum_{m \in l, p \in l}(e_m^r y_{jhm}^l + z_p^r s_{jhp}^l), \forall h \in l, r \in [R]$, where $m \in l, p \in l$ specify that schedule *l* trains the model using type-*m* workers and type-*p* PSs, and y_{jhm}^l (s_{jhp}^l) represents the given number of workers *m* (PSs *p*) on server *h* in *l*.

Constraint (8a) is equivalent to (2h). Constraint (8b) ensures that each job is executed according to at most one schedule. A feasible solution to ILP (8) has a corresponding feasible solution in problem (7), and vice versa, with the same objective value. Note that we introduce an exponential number of variables in ILP (8), each corresponding to a possible schedule of job *j*. To solve ILP (8), we formulate the dual LP of ILP (8) by relaxing $x_{jl} \in \{0, 1\}$ to $x_{jl} \ge 0$ and introducing dual variables $\delta_h^r(t)$ and u_j to constraints (8a) and (8b):

$$\text{minimize} \sum_{j \in [J_i]} u_j + \sum_{t \in [\tau_i]} \sum_{h \in [H]} \sum_{r \in [R]} \delta_h^r(t) C_h^r \tag{9}$$

subject to:

$$u_j \ge w_j - \sum_{t \in T(l)} \sum_{h \in l} \sum_{r \in [R]} \delta_h^r(t) f_{jh}^r(l), \forall j \in [J_i], l \in \Gamma_j, \tag{9a}$$

$$\delta_h^r(t), u_j \ge 0, \forall j \in [J_i], t \in [\tau_i], h \in [H], r \in [R]. \tag{9b}$$

If we interpret dual variable $\delta_h^r(t)$ as the unit cost of type-*r* resource on server *h* in time *t*, then $\sum_{t \in T(l)} \sum_{h \in l} \sum_{r \in [R]} \delta_h^r(t) f_{jh}^r(l)$ is the total resource cost of all workers and PSs serving job *j* by schedule *l*. The RHS of (9a), *i.e.*, job weight minus overall resource cost of job *j* with schedule *l*, is the job utility. To minimize the dual objective, we assign dual variables u_j to be the maximum between 0 and the RHS of (9a) according to the best schedule l_j:

$$u_j = \max\{0, \max_{l \in \Gamma_j} \text{RHS of (9a)}\}. \tag{10}$$

If $u_j > 0$, we construct schedule of job *j* according to l_j ($x_{jl_j} = 1$); or otherwise, we do not schedule it ($x_{jl} = 0, \forall l \in \Gamma_j$). The rationale is that, given limited resources, we wish to schedule jobs with larger utility.

$A_{maxweight}$ in Algorithm 3 is our offline algorithm for the maximum weighted schedule problem with the input job set ϕ. Line 1 initializes primal and dual variables. For each job *j* in ϕ, lines 3 and 4 invoke $A_{mincost2}$ and $A_{mincost1}$ to find a schedule with the lowest cost in the two cases, *i.e.*, $q_j = 1$ and $q_j = 0$, respectively. Comparing the resulting solutions, we obtain the best schedule with the highest utility u_j for job *j* in lines 5-7. If $u_j > 0$, we set all primal variables according to l_j in lines 10-11 and update the dual variables using the following carefully designed price functions $\delta_h^r(\cdot)$ in line 14. Line 12 updates J_i^s, *i.e.*, the set of jobs which have been scheduled in the *i*th round. In line 13, $\beta_h^r(t)$ records the amount of allocated type-*r* resource on server *h* for time *t*.

$$\delta_h^r(\beta_h^r(t)) = \lambda^{\frac{\beta_h^r(t)}{C_h^r}} - 1, \forall h \in [H], r \in [R], t \in [\tau_i], \quad \text{where } \lambda = 2(THRF) + 1 \tag{11}$$

We make two assumptions. First, we assume that a job's weight is proportional to its resource consumption, *i.e.*, $1 \le \frac{w_j}{\sum_{t \in T(l)} \sum_{h \in l} \sum_{r \in [R]} f_{jh}^r(l)} \le F, \forall j, l, h, r$. Here parameter F represents the upper bound of a job's weight to its resources consumption, and it will be used to design the

Algorithm 3 Total Weight Maximization $A_{maxweight}$

Input: $\phi, \tau_i, C_h^r, \forall h \in [H], r \in [R]$;
Output: $x_{jt}, y_{jhm}, s_{jhp}, d_j, q_j, J_i^s, \forall j \in [J_i], t \in [\tau_i], m \in [M], p \in [P], h \in [H]$;
Initialize $x_{jt} = 0, d_j = 0, y_{jhm} = 0, s_{jhp} = 0, \beta_h^r(t) = 0, \delta_h^r(t) = \Delta_h^r(0), \forall j \in [\phi], t \in [\tau_i], m \in [M], h \in [H], p \in [P], r \in [R]$;
for each job $j \in [\phi]$ **do**
  $(\text{cost}_j, l_j) = A_{mincost2}(\tau_i, \{\beta_h^r(t)\}, \{\delta_h^r(t)\}, \{C_h^r\})$;
  $(\text{cost}, l) = A_{mincost1}(\tau_i, \{\beta_h^r(t)\}, \{\delta_h^r(t)\}, \{C_h^r\})$;
  if $\text{cost} < \text{cost}_j$ **then**
    $\text{cost}_j = \text{cost}, l_j \Leftarrow l$;
  end if
  $u_j = w_j - \text{cost}_j$;
  if $u_j > 0$ **then**
    $x_{jt^-} = 1, d_j = L_j$;
    Set q_j, y_{jhm}, s_{jhp} according to l_j, $\forall h \in l_j, m \in l_j, p \in l_j$;
    $J_i^s = J_i^s \cup \{j\}$;
    $\beta_h^r(t) = \beta_h^r(t) + f_{jh}^r(l_j), \forall t \in T(l_j), h \in [H], r \in [R]$;
    Update $\delta_h^r(t), \forall t \in T(l_j), h \in [H], r \in [R]$ with (11);
  end if
end for

price function of the unit resource. Second, $\frac{f_{jh}^r(l)}{C_h^r} \leq \frac{1}{\log \lambda}$, which implies that the one type resource demand of each job on one server is small as compared to the resource capacity of each server. The price function starts at zero and increases exponentially with the increase of resource consumption. When there is little usage of type-*r* resource on server *h*, $\beta_h^r(t)$ is close to zero, which allows jobs to consume resource freely. When type-*r* resource on server *h* is exhausted, $\beta_h^r(t)$ is close to the resource capacity C_h^r, and $\delta_h^r(t)$ grows fast to a carefully designed large value λ, so that type-*r* resource on server *h* will be barely allocated to a job, unless its weight is sufficiently large.

5.2 Cost Minimization Problem

Since w_j is a constant, the utility maximization problem of job *j* is equivalent to the following cost minimization problem:

$$\min \sum_{t \in [t', t'+d_j)} \sum_{h \in [H]} \sum_{r \in [R]} x_{jt'} \delta_h^r(t) \Big(\sum_{m \in [M]} e_m^r y_{jhm} + \sum_{p \in [P]} z_p^r s_{jhp} \Big) \tag{12}$$

subject to:
$$\sum_{t \in [\tau_i]} x_{jt} = 1, \tag{12a}$$

$$(7b), (2b) - (2g), (2i), (2k) - (2o), \forall t \in [\tau_i], \text{for the specific } j.$$

We next show the schedule that minimizes job *j*'s cost can be found efficiently and optimally using Algorithm 5 and Algorithm 4. When we fix the worker type m and the PS type p serving job *j*, the number of acquired time slots is at most $\lceil \frac{E_j D_j K_j}{\rho_{jm}^p} \rceil$. For a fixed allocated time slot d_j, the number of workers needed is at least $\lceil \frac{E_j D_j K_j}{d_j \rho_{jm}^p} \rceil$. If we further know the starting time of job *j*, problem (12) is simplified as the following ILP, where $m = m', p = p', t' = t^-, t^+ = t^- + d_j$:

$$\min_{\boldsymbol{y}, \boldsymbol{s}} \quad \text{cost}(m', p', t^-, t^+) = \sum_{t \in [t^-, t^+)} \sum_{h \in [H]} \sum_{r \in [R]} \delta_h^r(t) (e_{m'}^r y_{jhm'} + z_{p'}^r s_{jhp'}) \tag{13}$$

subject to:

$$q_j = 1 \text{ if and only if } h = h', \forall h, h' : y_{jhm'} > 0, s_{jh'p'} > 0, \tag{13a}$$

$$\sum_{h \in [H]} y_{jhm'} \leq D_j, \tag{13b}$$

$$\sum_{h \in [H]} y_{jhm'} \geq \lceil \frac{E_j D_j K_j}{d_j \rho_{jm'}^{p'}} \rceil, \tag{13c}$$

$$s_{jhp'} B_{p'} \geq \sum_{h' \in [H^{-h}]} y_{jh'm'} b_{m'}, \forall h : s_{jhp'} > 0, \tag{13d}$$

$$\sum_{h \in [H]} s_{jhp'} \geq 1, \tag{13e}$$

$$y_{jhm'}, s_{jhp'} \in \{0, 1, ...\}, \forall h \in [H], \forall p \in [P], \tag{13f}$$

$$q_j \in \{0, 1\}. \tag{13g}$$

That is, we need to find the best placement scheme for job *j* to minimize the overall resource cost satisfying constraints (13a)-(13g). Note that constraint (13d) is satisfied naturally, since the RHS of (13d) is zero. Besides, the processing capacity ρ_{jm}^p is affected by the location of workers and PSs. If all workers and PSs of one job are deployed in the same physical server, the bandwidth occupied by exchanging gradient/parameters can be ignored. Therefore, there are two cases according to whether all workers and PSs are deployed in the same server. For distributed ($q_j = 0$) and centralized placement ($q_j = 1$), deployment solutions of workers and PSs are different. We come up with algorithms to find the best schedule with the smallest cost for job *j* as $A_{mincost2}$ and $A_{mincost1}$. $A_{mincost2}$ handles the case where all workers and PSs of job *j* are running on one server, *i.e.*, $q_j = 1$, $\rho_{jm}^p = 1/(v_{jm} + U_j^p)$, and $A_{mincost1}$ solves the other, *i.e.*, $q_j = 0$, $\rho_{jm}^p = 1/(v_{jm} + U_j^p + \frac{2\pi_j}{b_m})$.

In $A_{mincost1}$, we record the amount of available type-*r* resource on server *h* at time slot *t* using $\omega_h^r(t)$ in line 2. Next, we enumerate the worker and PS types serving job *j* in line 3 and 4. Then, we traverse possible execution time and compute the number of workers needed in lines 5-6. Given starting time t^- in line 7, we sort servers for worker m' deployment in non-decreasing order of total resource cost $\sum_{t \in [t^-, t^+)} \sum_{r \in [R]} \delta_h^r(t) e_{m'}^r$ recorded by Ω_h in line 8. Then lines 9-33 maximally deploy workers starting from the cheapest server, respecting capacity constraint (2h), the required number of workers N_j in (13c) and bandwidth reservation constraints (13d). Specifically, we decide the number of workers and PSs in given server *n* in lines 14-22 in a greedy manner, *i.e.*, the maximum number of workers and PSs are placed satisfying (13d). If there are not enough workers or PSs, completing job *j* is infeasible (lines 25 and 26); otherwise, we compute the overall cost $\sum_{t \in [t^-, t^+)} \sum_{h \in [H]} \sum_{r \in [R]} \delta_h^r(t) (e_{m'}^r y_{jhm'} + z_{p'}^r s_{jhp'})$ (line 28).

Algorithm 4 Subroutine for Job j $A_{mincost1}$

Input: $\tau_i, \beta_h^r(t), \delta_h^r(t), C_h^r, \forall h \in [H], , r \in [R], t \in [\tau_i]$;
Output: l_j, cost_m;

1: Initialize $u_j = 0, l_j = \varnothing$, cost_m $= +\infty$;
2: $q_j = 0, \omega_h^r(t) = C_h^r - \beta_h^r(t), \forall h, r, t$;
3: **for** $m' = 1$ to M **do**
4:   **for** $p' = 1$ to P **do**
5:     **for** $L_j = \lceil \frac{EjK_j}{\rho_{jm'}^{p'}} \rceil$ to $\lceil \frac{EjD_jK_j}{\rho_{jm'}^{p'}} \rceil$ **do**
6:       $N_j = \lceil \frac{EjD_jK_j}{L_j\rho_{jm'}^{p'}} \rceil, \hat{N} = N_j$;
7:       **for** $t^- = 1$ to $\tau_i - L_j$ **do**
8:         List $h \in [H]$ in nondecreasing order of Ω_h, $t^+ = t^- + L_j$;
9:         **for** $n = 1, ..., H$ **do**
10:           $y_{jhm} = 0, s_{jhp} = 0, \forall m, p, h$;
11:           **for** $k = 1, ..., H$ **do**
12:             $\hat{y} = \min\{\min_{r\in[R], t\in[t^-, t^+)} \lfloor \frac{\omega_k^r(t)}{e_{m'}^r} \rfloor, \hat{N}\}$;
13:             $y_{jkm'} = \hat{y}$;
14:             **if** $k = n$ **then**
15:               **for** $g = 0$ to $\hat{y}$ **do**
16:                 $\hat{s} = \min_{r\in[R], t\in[t^-, t^+)} \lfloor \frac{\omega_n^r(t) - ge_{m'}^r}{z_{p'}^r} \rfloor$;
17:                 **if** $\hat{s}B_{p'} \geq (N_j - g)b_{m'}$ **then**
18:                   $y_{jnm'} = g$;
19:                   $s_{jnp'} = \min\{\hat{s}, \lceil \frac{(N_j - g)b_{m'}}{B_{p'}} \rceil\}$;
20:                 **end if**
21:               **end for**
22:             **end if**
23:             $\hat{N} = \hat{N} - y_{jkm'}$;
24:           **end for**
25:           **if** $\hat{N} > 0$ **or** $s_{jnp'} < 1$ **then**
26:             cost $= +\infty$;
27:           **else**
28:             Compute cost;
29:           **end if**
30:           **if** cost < cost_m **then**
31:             cost_m = cost, $l_j \Leftarrow \{t^-, L_j, \boldsymbol{y}, \boldsymbol{s}, q_j\}$;
32:           **end if**
33:         **end for**
34:       **end for**
35:     **end for**
36:   **end for**
37: **end for**
38: **return** l_j, cost_m

We identify the schedule with smallest cost in lines 30-32. Finally, we return the resulting schedule l_j and the corresponding cost cost_m in line 38.

Compared to $A_{mincost1}$, $A_{mincost2}$ counts the range of acquired time slots and number of workers needed with different processing capacities. We enumerate the server to run all workers and PSs on it.

5.3 Theoretical Analysis

Theorem 1. *Algorithm 4 and Algorithm 5 yield an optimal solution of problem (13) in two scenarios, respectively.*

Proof. Please see Appendix. □

Theorem 2. *$A_{maxweight}$ in Algorithm 3, with $A_{mincost2}$ and $A_{mincost1}$, computes a feasible solution to problems (7)(8)(9).*

Algorithm 5 Subroutine for Job j $A_{mincost2}$

Input: $\tau_i, \beta_h^r(t), \delta_h^r(t), C_h^r, \forall h \in [H], , r \in [R], t \in [\tau_i]$;
Output: l_j, cost_m;

Initialize $u_j = 0, l_j = \varnothing$, cost_m $= +\infty$;
$q_j = 1, \omega_h^r(t) = C_h^r - \beta_h^r(t), \forall h, r, t$;
while traverse the value space of variables m' p' L_j t^- in order **do**
  for $h = 1, ..., H$ **do**
    $y_{jhm} = 0, s_{jhp} = 0, \forall m, p, h$;
    Compute $y_{jhm'}$ and $s_{jhp'}$ respecting (2h) and (13a)
    Set cost according to the feasibility of $y_{jhm'}$ and $s_{jhp'}$
    if cost < cost_m **then**
      cost_m = cost, $l_j \Leftarrow \{t^-, L_j, \boldsymbol{y}, \boldsymbol{s}, q_j\}$;
    end if
  end for
end while
return l_j, cost_m

Proof. Please see Appendix. □

Theorem 3. *The approximation ratio of $A_{maxweight}$ in Algorithm 3 is $2\log\lambda$.*

Proof. Please see Appendix. □

Theorem 4. *A_{online} in Algorithm 1 runs in polynomial time, with time complexity $O((\log w(J))JMPT^2 \log T(H \log H + H^2))$.*

Proof. Please see Appendix. □

Theorem 5. *A_{online} in Algorithm 1 is 4α-competitive, where $\alpha = \lfloor \frac{\log w(J) - \log w_{min}}{1 + \log\log\lambda - \log(2\log\lambda - 1)} \rfloor + 1$, where λ are defined in (11), $w(J) = \sum_{j\in J} w_j$ and $w_{min} = \min_{j\in[J]} w_j$.*

Proof. Please see Appendix. □

We observe that the typical value of α is close to 4 in simulation studies. As shown by the proof of Lemma 2, the value of α in each round i should satisfy inequality (6). According to the definition of J_{i1}^*, we can set α to be $\lfloor \frac{\log w(J_i) - \log w_{min}}{\log\gamma - \log(\gamma - 1)} \rfloor + 1$ in simulations. Further, if $J_i^s = J_i$ for the specific ι, we can terminate the ith round iteration of A_{dual} and turn to the next round.

6 Extension to Ring-AllReduce framework

In this section, we consider the total weighted completion time minimization problem with Ring-AllReduce architecture. Sec. 6.1 model the distributed ML system. We show that the approximation algorithm $A_{maxweight}$ can also handle the Ring-AllReduce architecture, in Sec. 6.2. We design two algorithms, which act as subroutines of $A_{maxweight}$, in Sec. 6.3 to find the best schedule for each job in the two cases, respectively. Theoretical analysis is conducted in Sec. 6.4.

6.1 Training Process with Ring-AllReduce Architecture

With data parallelism and AllReduce architecture, each worker of a job trains the entire model using different data chunks. The major computation steps on each worker are: (i) compute the gradient using a mini-batch; (ii) compute the mean of the gradients generating on all workers and return the resultant gradient to all other workers. This process is called *AllReduce*; (iii) update the model parameters.

There are several algorithms to implement the AllReduce operation, *e.g.*, Tree AllReduce, Round-robin AllReduce, Butterfly AllReduce and Ring-AllReduce. In this work, we focus on the Ring-AllReduce architecture. Ring-AllReduce eliminates the performance bottleneck by distributing the computation and communication over the participant workers. It has been more widely adopted than the others, given that it is efficient and simple to implement.

Considering a certain job j, let Φ be the total number of workers serving job j, *i.e.*, $\Phi = \sum_{h\in[H]}\sum_{m\in[M]} y_{jhm}$, and each worker is uniquely identified by a number $\varphi \in \Phi$. Let G_φ be the gradient of worker φ after training a mini-batch. First, each worker divides its own gradient into Φ parts. $G_{\varphi\kappa}$ is the κ-th part of G_φ. Let G_0 be the resultant gradient, whose size is the same as G_φ. The κ-th part of G_0 is to be: $G_{0\kappa} = G_{1\kappa}\ Op\ G_{2\kappa}\ Op\ \ldots\ Op\ G_{\Phi\kappa}$. Here Op is a binary operator. For example, the SUM operation is used to compute the mean of gradients in distributed deep learning. **First**, the worker φ sends $G_{\varphi\varphi}$ to the next worker $\varphi+1$, while it receives $G_{\varphi-1\varphi-1}$ from the previous worker $\varphi-1$ simultaneously. (The worker Φ sends $G_{\Phi\Phi}$ to the first worker, and vice versa.) That is, all workers constitute a single ring. **Second**, worker φ performs the reduction operation to the received gradient $G_{\varphi-1\varphi-1}$ and its own gradient $G_{\varphi\varphi-1}$, and sends the reduced gradient to the next worker $\varphi+1$. By repeating the receive-reduce-send steps $\Phi-1$ times, each worker obtains a different portion of the resulting gradient. Finally, all worker can obtain the completed gradient by sharing the distributed partial results among them.

In the Ring-AllReduce algorithm, we can calculate the amount of communication in each worker in the following way. In the earlier half of the algorithm, each worker sends gradients $\Phi-1$ times, whose total size is $\frac{\pi_j(\Phi-1)}{\Phi}$. Next, each worker sends partial resulting gradient $\Phi-1$ times of the same total size. Thus, the total amount of data each worker sends throughout the algorithm is $\frac{2\pi_j(\Phi-1)}{\Phi}$, which is practically independent of Φ. Similarly, the computation time at each worker for performing the reduction operation in the earlier half of the algorithm is $\frac{U_j(\Phi-1)}{\Phi}$, here U_j is the time to process gradients of size π_j at a worker. When $q_j = 1$, *i.e.*, all workers of j are deployed on one server, the communication time of each worker can be ignored. When $q_j = 0$, *i.e.*, workers of job j are placed on at least two servers, with synchronous training, the time for all workers to transfer gradients in one iteration is $\frac{2\pi_j(\Phi-1)}{\Phi b_m}$. Thus, we have

$$\rho_{jm} = \begin{cases} 1/(v_{jm} + \dfrac{U_j(\Phi-1)}{\Phi}), & \text{if } q_j = 1 \\ 1/(v_{jm} + \dfrac{U_j(\Phi-1)}{\Phi} + \dfrac{2\pi_j(\Phi-1)}{\Phi b_m}), & \text{if } q_j = 0 \end{cases} \tag{14}$$

With Ring-AllReduce architecture, the offline minimization problem is formulated as follows:

$$\text{minimize} \quad \sum_{j\in[J]} w_j \sum_{t\in[T]} x_{jt}(t+d_j) \tag{15}$$

subject to:

$$q_j = 1 \text{ if and only if } h = h', \forall h, h' : y_{jhm} > 0, y_{jh'm} > 0, \forall j, \tag{15a}$$

$$\sum_{j:t'\in(t-d_j,t]} x_{jt'} \sum_{m\in[M]} e^r_m y_{jhm} \le C^r_h, \forall t, \forall r, \forall h, \tag{15b}$$

$$(2a)(2b), (2f)-(2g), (2j)-(2k), (2m)-(2o).$$

Constraint (15a) shows the relationship among q_j and y_{jhm}. The resource capacity of physical servers for running workers is formulated in constraint (15b). Here, $x_{jt'} = 1, t' \in (t-d_j, t]$ denotes that job j is still running in time slot t.

Note that A_{online} and A_{dual} can be applied to both distributed ML architectures we described.

6.2 The Maximum Scheduled Weight Problem

The maximum scheduled weight problem for J_i by time τ_i in each iteration i is formulated as the following integer program:

$$\text{maximize} \quad \sum_{j\in[J_i]}\sum_{t\in[\tau_i]} w_j x_{jt} \tag{16}$$

subject to:

$$(7a), (7b), (15a), (15b), (2b), (2f)-(2g), (2k), (2m)-(2o),$$

$$\text{where } \forall t \in [\tau_i].$$

To address non-conventional constraints (2b)(2f)(7b), we reformulate problem (16) into an equivalent conventional ILP using the compact-exponential technique:

$$\text{maximize} \quad \sum_{j\in[J_i]}\sum_{l\in\Gamma_j} w_j x_{jl} \tag{17}$$

subject to:

$$\sum_{j\in[J_i]}\sum_{l:t\in T(l), h\in l} x_{jl} f^r_{jh}(l) \le C^r_h, \forall t \in [\tau_i], r \in [R], h \in [H], \tag{17a}$$

$$\sum_{l\in\Gamma_j} x_{jl} \le 1, \forall j \in [J_i], \tag{17b}$$

$$x_{jl} \in \{0,1\}, \forall j \in [J_i], l \in \Gamma_j. \tag{17c}$$

Here, each feasible schedule of job j $l \in \Gamma_j$ corresponds to the set of decisions $(x_{jt}, d_j, y_{jhm}, q_j, \forall m \in [M], h \in [H], t \in [\tau_i])$ satisfying constraints (7b)(15a)(2b)(2f)-(2g)(2k)(2m)-(2o). $f^r_{jh}(l) = \sum_{m\in l} e^r_m y^l_{jhm}, \forall h \in l, r \in [R]$. Constraint (17a) is equivalent to (15b). A feasible solution to ILP (17) has a corresponding feasible solution in problem (16), and vice versa, with the same objective value. To solve ILP (17) with an exponential number of variables, we formulate the dual LP of ILP (17) by relaxing $x_{jl} \in \{0,1\}$ to $x_{jl} \ge 0$ and introducing dual variables $\delta^r_h(t)$ and u_j to constraints (17a) and (17b):

$$\text{minimize} \sum_{j\in[J_i]} u_j + \sum_{t\in[\tau_i]}\sum_{h\in[H]}\sum_{r\in[R]} \delta^r_h(t) C^r_h \tag{18}$$

subject to:

$$u_j \ge w_j - \sum_{t\in T(l)}\sum_{h\in l}\sum_{r\in[R]} \delta^r_h(t) f^r_{jh}(l), \forall j \in [J_i], l \in \Gamma_j, \tag{18a}$$

$$\delta^r_h(t), u_j \ge 0, \forall j \in [J_i], t \in [\tau_i], h \in [H], r \in [R]. \tag{18b}$$

$\sum_{t\in T(l)}\sum_{h\in l}\sum_{r\in[R]} \delta^r_h(t) f^r_{jh}(l)$ is the total cost of resource occupied by job j with schedule l. The RHS of (18a) is the job utility, which equals to the job weight minus overall resource cost of all workers serving job j by schedule l. We minimize the dual objective by setting u_j to maximum of 0 and the RHS of (18a) with the best schedule l_j:

$$u_j = \max\{0, \max_{l \in \Gamma_j} \text{RHS of (18a)}\}. \tag{19}$$

If $u_j > 0$, we construct schedule of job *j* according to l_j ($x_{jl_j} = 1$); or otherwise, we do not schedule it ($x_{jl} = 0, \forall l \in \Gamma_j$).

Note that $A_{maxweight}$ can also handle the Ring-AllReduce architecture by using subroutines $A_{RAmincost1}$ and $A_{RAmincost2}$ in line 3 and 4 and keeping $s_{jhp}, \forall j \in [J], h \in [H], p \in [P]$ equals to 0 all the time.

6.3 Cost Minimization Problem

The utility maximization problem of job *j* is equivalent to the following cost minimization problem:

$$\min \sum_{t \in [t', t'+d_j)} \sum_{h \in [H]} \sum_{r \in [R]} x_{jt'} \delta_h^r(t) \sum_{m \in [M]} e_m^r y_{jhm} \tag{20}$$

subject to:

$$\sum_{t \in [\tau_i]} x_{jt} = 1, \tag{20a}$$

$$(7b), (15a), (2b), (2f) - (2g), (2k), (2m) - (2o), \forall t \in [\tau_i],$$

for the specific j.

We next show the schedule that minimizes job *j*'s cost can be found efficiently and optimally using Algorithm 6 and Algorithm 7. When we fix the worker type m and the number of workers serving job *j*, the number of acquired time slots is at least $\lceil \frac{EjD_jK_j}{\sum_{h\in[H]}\sum_{m\in[M]} y_{jhm}\rho_{jm}} \rceil$, *i.e.*, the total work time of all workers is at least $\lceil \frac{EjD_jK_j}{\rho_{jm}} \rceil$. If we further know the starting time of job *j*, problem (21) is simplified as the following ILP, where $m = m'$, $\sum_{h\in[H]}\sum_{m\in[M]} y_{jhm} = \Phi'$, $t' = t^-$, $t^+ = t^- + d_j$:

$$\min_{\boldsymbol{y},\boldsymbol{s}} \quad \text{cost}(m', t^-, t^+) \quad = \sum_{t \in [t^-, t^+)} \sum_{h \in [H]} \sum_{r \in [R]} \delta_h^r(t) e_{m'}^r y_{jhm'} \tag{21}$$

subject to:

$$q_j = 1 \text{ if and only if } h = h', \forall h, h' : y_{jhm} > 0, y_{jh'm} > 0, \tag{21a}$$

$$\sum_{h \in [H]} y_{jhm'} = \Phi', \tag{21b}$$

$$d_j \sum_{h \in [H]} y_{jhm'} \geq \lceil \frac{EjD_jK_j}{\rho_{jm'}} \rceil, \tag{21c}$$

$$y_{jhm'}, d_j \in \{0, 1, ...\}, \forall h \in [H], \tag{21d}$$

$$q_j \in \{0, 1\}. \tag{21e}$$

That is, we need to find the best placement scheme for job *j* to minimize the overall resource cost satisfying constraints (21a)-(21e). Similarly, consider the situation whether all *j*'s workers and PSs are deployed on the same server, *i.e.*, $q_j = 1$ or not *i.e.*, $q_j = 0$. We come up with algorithms to find the best schedule with the smallest cost for job *j* as $A_{RAmincost1}$ and $A_{RAmincost2}$. $A_{RAmincost1}$ handles the case where all workers of job *j* are running on one server, i.e., $q_j = 1$, $\rho_{jm} = 1/(v_{jm} + \frac{U_j(\Phi-1)}{\Phi})$, and $A_{RAmincost2}$ solves another, *i.e.*, $q_j = 0$, $\rho_{jm} = 1/(v_{jm} + \frac{U_j(\Phi-1)}{\Phi} + \frac{2\pi_j(\Phi-1)}{\Phi b_m})$.

In $A_{RAmincost1}$, we enumerate the worker types and the potential number of workers serving job *j* in line 3 and 4. And compute the execution time needed in lines 5. Given starting time t^- in line 6, we decide the deployment of workers in lines 8-19. More specifically, we enumerate the server to run all workers on it. Line 9 sets $\{y_{jhm}\}_{\forall h\in[H], m\in[M]}$ to zero, and uses $\omega_h^r(t)$ to record the amount of available type-*r* resource on server *h* at time slot *t*. Lines 10 calculates the number of workers respecting capacity constraint (2h), to fulfill job workload E_jD_j. If not enough workers can be deployed, completing job *j* is infeasible (lines 11 and 12); otherwise, we compute the overall cost $\sum_{t\in[t^-,t^+)}\sum_{h\in[H]}\sum_{r\in[R]} \delta_h^r(t) e_{m'}^r y_{jhm'}$ (line 14). We identify the schedule with smallest cost in lines 16-17. Finally, we return the resulting schedule l_j and the corresponding cost cost_m in line 23.

Algorithm 6 Subroutine for Job j $A_{RAmincost1}$

Input: $\tau_i, \beta_h^r(t), \delta_h^r(t), C_h^r, \forall h \in [H], , r \in [R], t \in [\tau_i]$;
Output: l_j, cost_m;
1: Initialize $u_j = 0, l_j = \varnothing$, cost_m $= +\infty$;
2: $q_j = 1$; /*deploy all j's workers on one server*/
3: **for** $m' = 1$ to M **do**
4:   **for** $\Phi' = 1$ to D_j **do**
5:     $\hat{d}_j = \lceil \frac{EjD_jK_j}{\Phi'\rho_{jm'}} \rceil$;
6:     **for** $t^- = 1$ to $\tau_i - \hat{d}_j$ **do**
7:       $t^+ = t^- + \hat{d}_j$;
8:       **for** $h = 1, ..., H$ **do**
9:         $y_{jhm} = 0, \omega_h^r(t) = C_h^r - \beta_h^r(t), \forall t \in [\tau_i], m \in [M], p \in [P], h \in [H], r \in [R]$;
10:         $y_{jhm'} = \min\{\min_{r\in[R], t\in[t^-,t^+)} \lfloor \frac{\omega_h^r(t)}{e_{m'}^r} \rfloor, \Phi'\}$;
11:         **if** $\Phi' > y_{jhm'}$ **then**
12:           cost $= +\infty$;
13:         **else**
14:           cost $= \sum_{t\in[t^-,t^+)} \sum_{r\in[R]} \delta_h^r(t) e_{m'}^r y_{jhm'}$;
15:         **end if**
16:         **if** cost < cost_m **then**
17:           cost_m = cost, $l_j \Leftarrow \{t^-, \hat{d}_j, \boldsymbol{y}, q_j\}$;
18:         **end if**
19:       **end for**
20:     **end for**
21:   **end for**
22: **end for**
23: **return** l_j, cost_m

$A_{RAmincost2}$ counts the maximum number of time slots with different processing capacity, *i.e.*, $\rho_{jm} = 1/(v_{jm} + \frac{U_j(\Phi-1)}{\Phi} + \frac{2\pi_j(\Phi-1)}{\Phi b_m})$, which is related to the type and number of workers, in line 5. Lines 9-13 maximally deploy workers starting from the cheapest server, respecting capacity constraint (2h) and the total number of workers Φ' in (21b). Line 14 verifies the feasibility of the solution and line 17 calculates the cost of feasible solution. Results are returned in line 25.

6.4 Theoretical Analysis

Theorem 6. *Algorithm 6 and Algorithm 7 producess optimal solution of problem (16) in two cases, respectively.*

Proof. Please see Appendix. □

Algorithm 7 Subroutine for Job j $A_{RAmincost2}$

Input: $\tau_i, \beta_h^r(t), \delta_h^r(t), C_h^r, \forall h \in [H], , r \in [R], t \in [\tau_i]$;
Output: l_j, cost_m;

Initialize $u_j = 0, l_j = \varnothing$, cost_m $= +\infty$;
$q_j = 0$; /*deploy j's workers on at least two servers*/
for $m' = 1$ to M **do**
  for $\Phi' = 1$ to D_j **do**
    $\hat{d}_j = \lceil \frac{E_j D_j K_j}{\Phi' \rho_{jm'}} \rceil$, $\hat{N} = \hat{d}_j$;
    for $t^- = 1$ to $\tau_i - \hat{d}_j$ **do**
      $t^+ = t^- + \hat{d}_j$, $\Omega_h = \sum_{t \in [t^-, t^+)} \sum_{r \in [R]} \delta_h^r(t) e_{m'}^r$;
      List $h \in [H]$ in nondecreasing order of Ω_h;
      for $h = 1, ..., H$ **do**
        $\omega_h^r(t) = C_h^r - \beta_h^r(t), \forall t \in [\tau_i], h \in [H], r \in [R]$;
        $y_{jhm'} = \min\{\min_{r \in [R], t \in [t^-, t^+)} \lfloor \frac{\omega_k^r(t)}{e_{m'}^r} \rfloor, \hat{N}\}$;
        $\hat{N} = \hat{N} - y_{jhm'}$;
      end for
      if $\hat{N} > 0$ **then**
        cost $= +\infty$;
      else
        cost $= \sum_{t \in [t^-, t^+)} \sum_{h \in [H]} \sum_{r \in [R]} e_{m'}^r y_{jhm'} \delta_h^r(t)$;
      end if
      if cost $<$ cost_m **then**
        cost_m $=$ cost, $l_j \Leftarrow \{t^-, \hat{d}_j, \boldsymbol{y}, q_j\}$;
      end if
    end for
  end for
end for
return l_j, cost_m

Theorem 7. *$A_{maxweight}$ in Algorithm 3, with $A_{RAmincost1}$ and $A_{RAmincost2}$, computes a feasible solution to problems (16)(17)(18).*

Proof. Please see Appendix. □

Theorem 8. *The time complexity of A_{online} in Algorithm 1 for Ring-AllReduce architecture is polynomial.*

Proof. Please see Appendix. □

Note that the approximation ratio of $A_{maxweight}$ for Ring-AllReduce architecture is the same as we claimed in Theorem 3. And the competitive ratio of A_{online} for Ring-AllReduce architecture is the same as we claimed in Theorem 5.

7 Performance Evaluation

Settings. We simulate an ML cluster running for $T \in [100, 300]$ time slots (default value: 150). Each time slot is one hour long. The default number of servers is 150. The overall resource capacities, **C**, are set to be approximately $[0.2, 0.5]$ fraction of the respective overall job resource demand, which is computed by adding the ideal resource demand of all jobs. Resources configuration of each server is set according to Amazon EC2 GPU instances P3, P2 and G3. The numbers of worker and PS types are set to be 8 and 10, respectively. Following similar settings in [3][4][2], we set resource configuration for each type worker as follows: 1 to 4 GPUs, 1 to 16 vCPUs and bandwidth of 100Mbps to 5Gbps. Resource configuration for each type PS is: 1 to 16 vCPUs and bandwidth of 5Gbps to 20 Gbps. For each job, w_j is in $[200, 5000]$, E_j is set within $[50, 100]$, D_j is in $[5, 50]$, K_j is in $[10, 50]$, U_j^p is in $[10, 100]$ milliseconds, v_{jm} is in $[0.001, 0.05]$ time slots, and π_j is within $[30, 575]$MB [37][4].

Algorithms for comparison. We compare A_{online} with three job scheduling policies: (i) FIFO: default scheduler in Hadoop and Spark [45]; jobs run by order of arrival, with fixed numbers and resource configuration of workers (and PSs). The number of workers is fixed to a number within $[1, 30]$ for FIFO. (ii) Dominant Resource Fairness Scheduling (DRF): default scheduler in YARN [8] and Mesos [7]; the numbers of workers (and PSs) are computed to achieve max-min fairness in dominant resources [6]. (iii) AntMan [17]: a cluster scheduler, which introduces two types of jobs: opportunistic job and resource-guarantee job. AntMan schedules resource-guarantee jobs first and allocates sufficient GPU resources to them. For opportunistic jobs, AntMan aims to utilize free resources to the best of its ability. Resource-guarantee jobs that suffer long queuing delay will be automatically executed as opportunistic jobs. (iv) Tiresias [20]: a preemptive scheduler, which aims to minimize the average JCT (*i.e.*, time from job submission to job completion).Tiresias assigns jobs according to the multiplication of a job's remaining workload and the number of resources, (*e.g.*, GPUs, RAM and CPUs). In (i)-(iv), the resource configuration of workers (and PSs) is the same as that in the ideal case, which is derived according to recent literature [10] [11] [12] in our simulation studies. We compare $A_{maxweight}$ with an algorithm from recent literature [44] which proposes a greedy strategy to schedule jobs with deadlines in the offline scenario.

7.1 Performance of A_{online}

1) Objective Value (PS framework): Fig. 2 compares the total weighted completion time produced by different algorithms under different numbers of jobs, where $T = 300$. A_{online} performs at least 30% better than the other algorithms in both cases. The objective value may grow with the increase of number of servers according to Fig. 3. Note that λ in price function (11) increases in line with the number of servers H. A_{online} prefers to schedule jobs of larger weight with larger λ when available resources are insufficient. Thus, when the overall resource capacities nearly remain the same, the total amount of fragment resources increases and effective resource capacity of the servers decreases with larger H. The objective values in Fig. 2 (Fig. 3) are the average of multiple trials. In Fig. 2 and Fig. 3, the total weight job completion time obtained by AntMan and Tiresias are both much larger than other algorithms. This is because job execution duration is lengthened due to frequent preemption. Fig. 4 calculates the objective value obtained by A_{online} under different F, *i.e.*, the upper bound of a job's weight to its resources consumption. Recall that parameter λ in the price function and the theoretical competitive ratio are related to F. We can see that for larger values of F, the objective value is larger. Larger F represents larger weights of served jobs, *i.e.*, jobs with weight which is not large enough will be executed later.

(*Ring-AllReduce framework*): Fig. 5 and Fig. 6 represent the total weighted completion time achieved by five algorithms

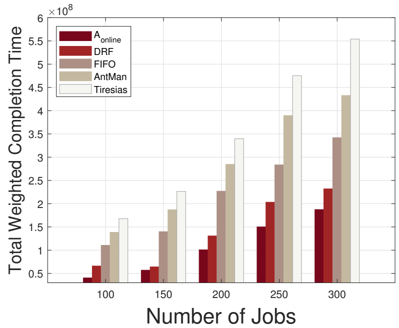

Fig. 2: Total weighted completion time with PS framework.

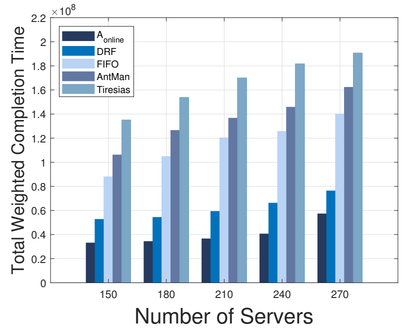

Fig. 3: Total weighted completion time with PS framework.

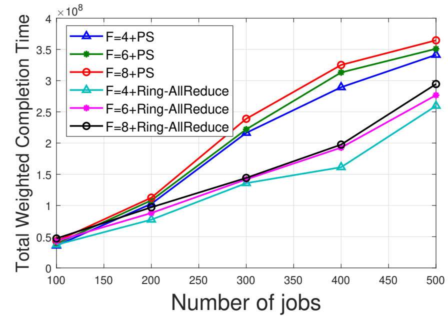

Fig. 4: Total weighted completion time of A_{online} under different F.

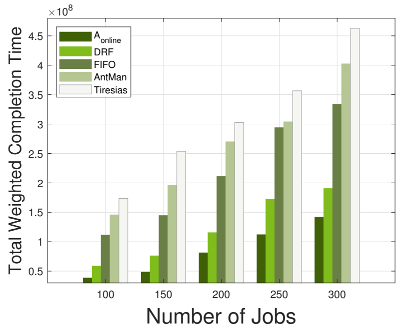

Fig. 5: Total weighted completion time with RA framework.

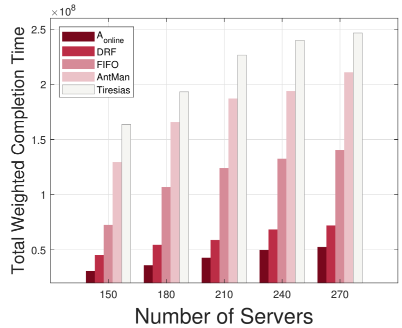

Fig. 6: Total weighted completion time with RA framework.

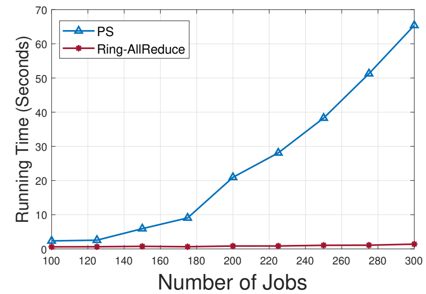

Fig. 7: Running time of A_{online} with different ML distributed system architectures.

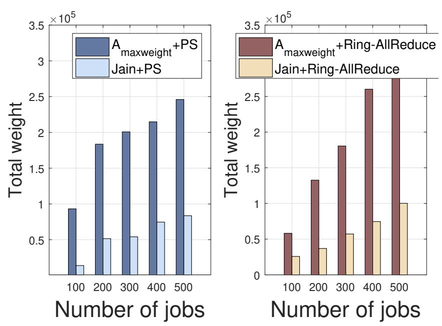

Fig. 8: Total scheduled job weight of $A_{maxweight}$ and Jain *et al.*'s algorithm [44].

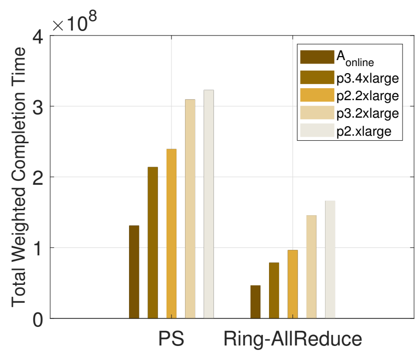

Fig. 9: The impact of workers' types.

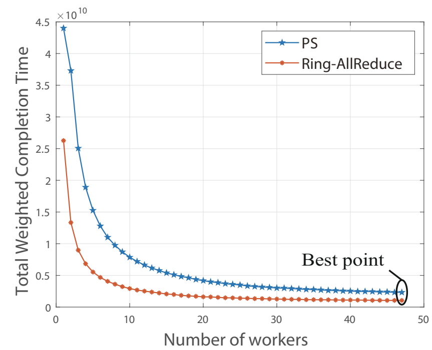

Fig. 10: The impact of the number of workers.

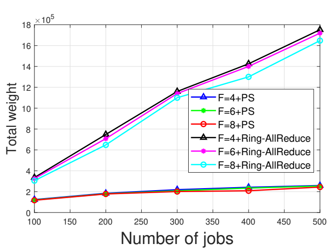

Fig. 11: Total weight of $A_{maxweight}$ under different F.

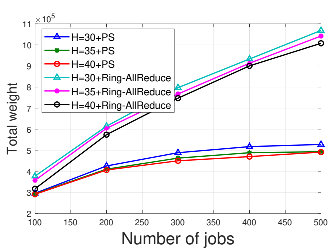

Fig. 12: Total weight of $A_{maxweight}$ under different H.

under different numbers of jobs and servers, respectively. And our online job scheduling algorithm A_{online} performs the best in both two cases. Compared to Fig. 2 and Fig. 3, the results are similar, which illustrates that A_{online} outperforms four baselines. In Fig. 6, we can observe that the total weighted completion time increases as the increase of number of servers, which is similar to the PS framework.

2) Running Time: We apply the *tic* and *toc* functions in MATLAB to measure the execution time of our online algorithm. We run 10 tests on a desktop computer (Intel Core i3-6100/8GB RAM) and present the average result in Fig. 7. We can observe that, the running time of A_{online} increases with the number of jobs, but still remains at a low level ($<$ 2 minutes).

3) The impact of demand elasticity: Fig. 9 compares the total weighted job completion time obtained by PS and Ring-AllReduce framework under different types of workers. To illustrate the impact of the types of workers, we select four frequent-used types of Amazon EC2 instances [40] (*i.e.*, p3.4xlarge, p2.2xlarge, p3.2xlarge and p2.xlarge) to act as different types of workers. Here, we assume that all jobs employ the same type of workers. The deployment of workers and PSs (including the number of workers/PSs and the execution time window) are determined by A_{online}. In Fig. 9, we can observe that different types of workers greatly affect the total weighted job completion time. A_{online} explores the demand elasticity, and chooses the best worker type for each job. Therefore, A_{online} can achieve the smallest total weighted job time. To investigate the impact of the number of workers, we plot the computing process of the number of workers in A_{online}. A_{online} enumerates all possible numbers and always selects the one with the smallest total weighted job completion time. Fig. 10 illustrates that the total weighted job completion time decreases as the increase of number of workers deployed for jobs. This is because the more workers allocated to the job, the faster the job would be completed.

7.2 Performance of $A_{maxweight}$

Fig. 8 compares the total weight achieved by $A_{maxweight}$ with related algorithm from recent literature [44]. Our offline algorithm $A_{maxweight}$ performs much better than the other. Fig. 11 represents the total weight of $A_{maxweight}$ under different F, which is related to price function in line 14 of $A_{maxweight}$. We can see that for smaller values of F, the total weight is larger. Smaller F represents more jobs can be served with the same total number of jobs, particularly, jobs with smaller weight. Fig. 12 shows the total weight of $A_{maxweight}$ under different H, *i.e.*, the number of servers to deploy workers and PSs. It reflects that the total weight is smaller for larger values of H because the total amount of fragment resources increases with the increase of the number of servers. In Fig. 12, there is an upward trend in the total weight with the increment of the number of jobs.

8 Conclusion

We proposed an online algorithm for scheduling synchronous training jobs in ML clusters. The online algorithm targets total weighted completion time minimization, consisting of (i) an online greedy-interval algorithm that converts the online scheduling problem into a series of batch processing problems; (ii) a primal-dual algorithm running for each batch, which computes the best execution window of each job, with proper number and type of workers (and parameter servers). Both theoretical analysis and trace-driven simulation studies validate our online algorithm's good performance, as compared to both offline optimum and commonly used scheduling algorithms in read-world cloud systems.

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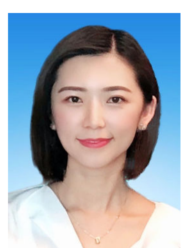

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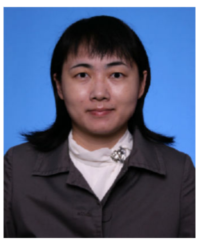

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